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Suppressing the Segregation of Granular Mixtures in Rotating Tumblers

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While an understanding of the dynamics of segregation has begun to emerge, controlling segregation continues to be a complicated problem. The use of time-modulation—via selective baffle placement—in order to suppress segregation in rotating tumblers is explored. Bidisperse (size or density), cohesionless granular materials in quasi-two-dimensional (2-D) rotating containers are studied by means of simulations and experiments. Results are presented in two main configurations for the placement of the baffles, (1) axial placement, and (2) attached to the periphery of the tumbler. Both experiments and simulations indicate that baffles attached at the periphery are ineffective in reducing segregation, while baffles axially located in the tumbler generate periodic flow inversions and dramatically reduce both density and size segregation. Qualitative and quantitative evidence is presented, in terms of the intensity of segregation. Theoretical and scale-up arguments are provided for the practical implementation of this approach. © 2008 American Institute of Chemical Engineers *AIChE J*, 54: 3124–3132, 2008
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Introduction

Segregation in granular materials has been the subject of study for a long time,[1–4] but its theoretical understanding even in the most simple cases is still far from complete. Small differences in almost any mechanical or surface property invariably lead to flow-induced pattern formation,[5,6] layering,[7,8] or complete separation of the materials.[9–11] Segregation is known to cause numerous problems and revenue losses during handling, processing, or manufacturing of particulate materials. If one rotates a mixture (with differences in size or density) of particles in a cylinder, it usually results in rapid radial segregation with the smaller (or denser) particles concentrating in a central core, and the larger (lighter) particles in the periphery. Recently, several approaches have been proposed with the purpose of controlling segregation, including cohesive manipulation[12,13] or particle modification (to *balance* competing segregation modes);[14,15] however, these techniques are not robust to changes in particle properties and/or cohesion degree, and, therefore, are applicable to only a subset of typical applications. For a review of other methods that have been used to prevent, reduce, or eliminate segregation of various types, the reader is referred to Ref. 16.

It is well established that mixing and segregation patterns are sensitive to the container geometry and fill level.[17–19] A relatively common way of mixing particles in several processing applications is through the use of partially filled rotating tumblers, where baffles are often used to augment mixing. Relatively little is known, however, from a theoretical point of view about the effect of baffles on solid mixing, even in cases limited to monodispersed systems. Recently, Shi et al.[20] have shown that periodic flow inversions via selective baffle placement—in a tumbler-type mixer—can serve as a generic method for eliminating segregation in this type of free-surface flow.[21–23] In this article, we use experiments and simulations to study the effect of selective baffle placement on mixing of binary mixtures with different sizes or densities. We will show by way of experiments and simulations that baffles attached at the periphery of a tumbler—probably the most common configuration in industrial practice—are ineffective in reducing segregation.

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Experimental

The experimental apparatus consists of a half-filled tumbler with a mixture of particles with different size or density, respectively. Experiments are conducted using 50/50 volume fraction combinations of cellulose acetate and/or glass beads. The tumbler is rotated around the horizontal axis with constant rotation rate of 6 RPMs, using a computer-controlled stepper motor. A glass cover provides access to the contents for imaging. Images of the tumbler are taken with a Nikon D200 digital camera with a resolution of 6MPixels, every half of a rotation. All experiments run for a total of 20 rotations. Previous experimental observations have shown, however, that segregation patterns reach steady state at approximately 4-5 rotations. Size segregation experiments involve 1.5:1 ratios (3 mm:2 mm) of cellulose acetate particles, while density segregation experiments use 2 mm glass and 2 mm cellulose acetate particles, with densities of 2,500 kg/m^3 and 1,240 kg/m^3, respectively. The ratio of the tumbler radius to bead diameter is in the range 18 to 50.

In a typical experimental run, particles are placed in the tumbler in a completely segregated initial state. Images are captured of the segregation/mixing patterns every half of a rotation. The RGB-digitized images of the experiment are transformed from the RGB color-space into the HSI color-space. The transformation used for calculating the HSI values follows the procedure described by Peterson et al.[24] Later, image analysis techniques are used in the thresholding and binning of the different particles. This procedure yields the information required in the calculation of intensity of segregation for each experimental run.

The dynamics of segregation for a rotating tumbler in the rolling regime are analyzed by the advection of the particles by the flow. Particles of different densities or sizes are initially placed in two halves of the drum, and their distribution after a given mixing time is analyzed. Particles in the passive region of the drum undergo solid-body rotation; particles in the cascading layer experience linear motion due to shear, and diffusion-like (random) motion takes place due to interparticle collisions. The average velocity in the cascading layer determines the residence time of the particles in the layer relative to the passive region. The difference in circulation times as a function of radial position results in convective mixing of the particles in unbaffled circular cylindrical tumblers. The diffusive mixing in the bed is determined by both the diffusivity in the shear layer, as well as the circulation time—which determines the number of passes through the shear layer. Several different measures exist that can be used to quantify the degree of mixedness, including among others the intensity of segregation,[9] and the mixing index.[25] The intensity of segregation calculated from Eq. 1 is used here to asses the effect of different baffle configurations on the segregation of mixtures with differences in density or size. The intensity of segregation (I_s) is calculated from

$$I_s = \left[\frac{1}{N-1} \sum_{i=1}^{N} (C_i - \langle C \rangle)^2 \right]^{1/2} \quad (1)$$

where C_i are concentrations at a set of N uniformly distributed points in the bed. The concentration at a given point is calculated as the fraction of one type of particle in a box of specified size centered on the point.

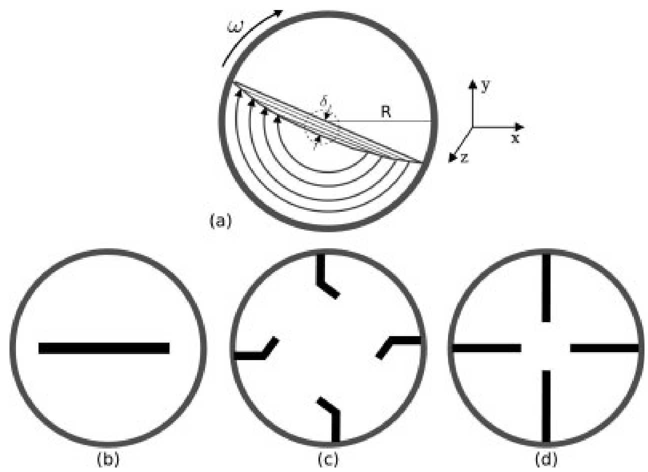

Figure 1. (a) Schematic of a typical rotating cylinder with no baffles, showing the definition of the shear layer thickness δ.

The shear layer defines the region within the bed where the main motion of the particles is taking place, and defines the boundary from the region of solid-body rotation. Representative streamlines, the coordinate system and the relevant system parameters are also shown. (b)–(d), Three configurations of baffled tumblers used in experiments and simulations. Note that these three configurations provide different ways of modifying the flow—and, therefore, the mixing—inside a tumbler.

Figure 1a illustrates the expected flow field in a half-filled tumbler with no baffles. The 2-D cylinder with radius R rotates at constant angular velocity ω, particles below the free surface move in solid-body rotation, while near the free surface particles move relative to each other in a lens shaped shear region designated as the shear or flowing layer. Figure 1b–d depicts three of the baffled configurations used both in experiments and simulations.

Particle dynamics

In the particle dynamics (PD) technique the bulk flow of the material is captured via simultaneous integration of the interaction forces between individual pairs of particles.[26,27] While these forces typically include only contact forces—normal (Hertzian) repulsion and tangential (Mindlin) friction (see Ref. 28)—and gravity, additional particle interaction forces (such as surface adhesion,[29,30] and van der Waals,[31] etc.) can be easily added. In particular, this technique has been quite successful in simulating granular materials, yielding insight into such diverse phenomena as force transmission,[32] wave propagation,[33] agglomeration formation and breakage,[29] cohesive mixing,[34] bubble formation in fluidized beds,[35] and segregation of free-flowing materials.[9]

In a granular flow, the particles experience forces due to interactions between particles (e.g., collisions, contacts, or cohesive interactions), as well as interactions between the system and the particles (e.g., gravitational forces). In this work, the collisional forces are modeled after the work of Hertz and Mindlin.[28] A thorough description of the interaction laws from contact mechanics (collision forces) can be found in References 36–38; therefore, they will not be reviewed here.

Figure 1a illustrates the coordinate system and the numerical setup used in the simulation of the rotating tumbler. The numerical experiments consist of a bidisperse system of perfectly smooth spheres bounded by a wall of immobile particles with periodic boundaries in the z direction. The number of particles is determined by the fill level and the particle diameters. The wall of the drum is rotated at a constant angular velocity ω. A typical initial condition for the rotating tumbler simulations is obtained by allowing a bed of particles arranged in a randomly perturbed lattice to settle under the action of gravity. From this initial configuration, a prescribed angular velocity is imposed and the simulation is allowed to proceed for approximately 20 revolutions based on the rotation rate prescribed. Then, the intensity of segregation (Eq. 1) rate of mixing are determined as a function of time.

Both the (PD) simulations and the experiments use glass and cellulose acetate particles in thin ($\approx$6–7 particle diameter) $\approx$70 particle wide tumblers that are rotated at 6 RPMs. The simulated particle sizes and densities (as well as vessel size) are matched to their corresponding experimental values; the particle stiffness used is reduced in order to decrease necessary simulation time (a practice shown to have essentially no impact on flow kinematics[27]).

Time modulation and free surface flows

In tumblers and other surface-dominated flows, both experiments and simulations have shown that segregation occurs only in the flowing layer; particles underneath this layer are effectively locked in place until they re-enter the flowing layer. Taking here density segregation as an example, under steady-flow conditions (i.e., in the rolling regime), dense particles migrate downward, perpendicular to the direction of the mean flow. In just a few passes through the flowing layer, the dense particles form a centrally-located, unmixed core.[39]

Time-modulation in fluid mixing and other dynamical systems[40] is a common practice, but has found only limited application in granular processing.[17,39,41] Moreover, until recently,[20] its utility in granular systems has been limited exclusively to increasing mixing rates. The key to adapting this idea to limiting to extent of free-surface segregation lies in recognizing that it takes a finite time for material to segregate, and that there is always a preferred direction that particles tend to segregate. In order to exploit these two facts, one needs to perturb the flow at a sufficiently high frequency f, such that $f > t_s^{-1}$, where t_s is the characteristic segregation time.

A critical issue with this technique is that a full understanding of segregation kinetics—and, therefore, the characteristic segregation time t_s—is still lacking. Nevertheless, using existing theoretical tools,[9,42] an estimate of the value of t_s, and, therefore, the critical forcing frequency f_{crit}, may be obtained via scaling arguments. One may write a segregation flux expression as $J_s = v_s\phi$ where v_s is the segregation velocity, and ϕ is the concentration of the segregating species. For density segregation, the segregation velocity will take the form $v_s = K_s(1-\bar{\rho})$,[9] where $(1-\bar{\rho})$, is the dimensionless density difference, and K_s will depend on the local void fraction and granular temperature. The characteristic segregation time may then be written as

$$t_s = R/[K_s(1-\bar{\rho})], \tag{2}$$

so that

$$f_{crit} = \frac{K_s(1-\bar{\rho})}{R}, \tag{3}$$

where R is the radius of the particles. A (low) estimate of the effective forcing frequency within a tumbler-type mixer may be obtained in the following way. Presuming that there is one axially-located baffle within the drum then, by symmetry, the flowing layer will be "interrupted" and have the segregation orientation altered once per half revolution. For a half-filled drum, this means that the material will pass through the flowing layer, on average, once prior to interacting with a baffle. If we use the residence time of particles within the flowing layer and multiply it by the number of layer passes prior to re-orientation, we obtain the inverse of the effective forcing frequency. Using the value of the mean residence time of the particles in the layer for an *unbaffled* tumbler, given as $\tau_{mean} = 2\pi/\sqrt{\omega\gamma}$ (where ω is the rotation rate, and γ is the shear rate in the flowing layer),[43] we get

$$f_{eff} = \frac{1}{\tau_{mean}} = \frac{\sqrt{\omega\gamma}}{2\pi}. \tag{4}$$

We suggest that this leads to a low estimate of the effective forcing frequency primarily because the layer will be effectively truncated by the baffle, such that we expect the residence time in the layer to be shortened relative to an unbaffled case. Combining Eqs. 3 and 4 leads to a ratio of the effective forcing frequency to that of the critical forcing frequency given as

$$\frac{f_{crit}}{f_{eff}} = \frac{2\pi K_s(1-\bar{\rho})}{\sqrt{\omega\gamma}R} \tag{5}$$

Previous experimental and computational results suggest that the parameter $(K_sR)/D$, is a decreasing function of the fluctuation energy of the flow and should be close to unity at small to moderate flow energies.[9] Combining this observation with the Savage[44] relation for diffusivity $D \propto R^2\gamma$, we can rewrite our dimensionless forcing frequency as

$$\frac{f_{crit}}{f_{eff}} = \xi(1-\bar{\rho})\sqrt{\frac{\lambda}{\omega}}, \tag{6}$$

where, due to current theoretical uncertainty and the time-varying nature of our flows (as well as our granular temperature, pressure, etc.), ξ is treated as a fitting parameter that is a weak function of the local number density, the granular temperature and the composition of the mixture.[9] The impact of this expression on tumbler design will be discussed later in the section on scaling.

Results

In this section we examine segregation experiments and simulations in half-filled tumblers, for both density (Figures

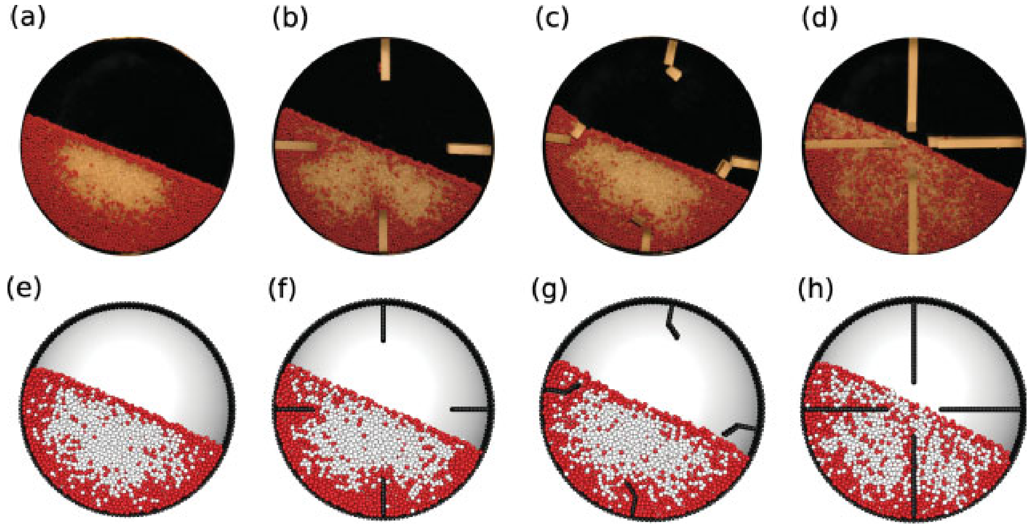

Figure 2. Asymptotic mixing results in horizontal drum (tumbler) mixers with baffles at the wall for both experiments (Top), and simulations (Bottom).

(a)–(e) Unbaffled tumbler, (b)–(f) short baffle, (c)–(g) short baffle with angled tip—a traditional configuration, and (d)–(h) long baffles.

2–5) and size (Figures 6–9), in two main configurations for the placement of the baffles: (1) axial placement in the tumbler, and (2) attached to the periphery of the cylinder. In industry, baffles are often used to augment mixing with an implied reduction in segregation; however, in what follows we will show that baffles attached at the periphery of a tumbler, are for the most part, ineffective in reducing segregation. In all cases, the simulated particle sizes and densities (as well as vessel size) are matched as closely as possible to their corresponding experiments.

Density segregation

We begin by considering density segregation in tumblers with baffles attached to the walls, a very common configuration in industrial practice. Figure 2 depicts a qualitative comparison of the asymptotic mixing for both experiments and simulations in three configurations of baffle placement at the walls. In columns left to right, Figure 2 presents images of experiments and simulations for a tumbler with no baffles, short baffles, short baffles with angled tips, and long baffles. Note that we define long baffles as those that actually transversely cut a portion of the flowing layer, much like the axially oriented baffles, as discussed later.

Both experiment and simulations show segregation with the formation of a core of dense particles (white, corresponds to glass particles) for three of the four configurations, only the long baffle case exhibit reduced segregation. Turning toward quantitative measures of these observations, Figure 3 shows a plot of the intensity of segregation as a function of time for the four configurations in Figure 2. The results show that while the short traditional baffles produce results similar to the nonbaffled case, the long baffle reduces the measured asymptotic degree of segregation, although this configuration is prone to "choke" the flow due to the small clearance between the baffles.

The qualitative comparison for axially-placed baffles is illustrated in Figure 4. From top to bottom, Figure 4 presents images of experiments and simulations for a tumbler with no baffles, one axial baffle, and three axial baffles.

The results in Figure 4 show the expected observation of core formation in the case of the unbaffled tumbler. Both experiment and simulations show that for axially-placed baffles the segregation is dramatically reduced. Furthermore, the quantitative data in Figure 5 confirm these qualitative observations, and indicate that placing an axial baffle results in significantly lower IS values, and, therefore, a better degree of mixing.

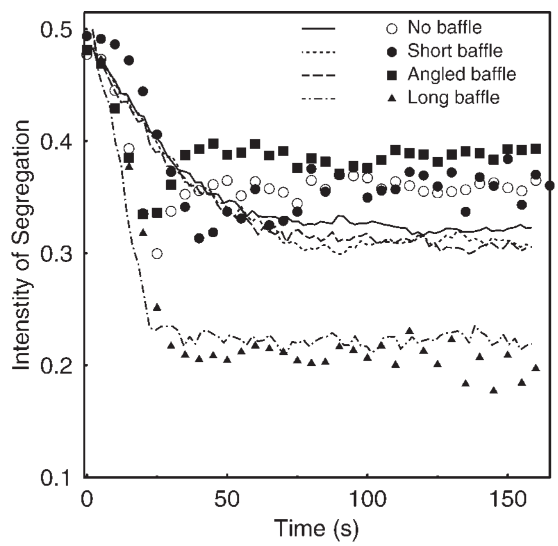

Figure 3. Quantitative mixing results for density segregation in tumbler mixers with baffles attached to the walls.

Experiments are shown as symbols and simulations as lines. Note that unbaffled mixers and mixers with short baffles behave very similarly, while very long traditional baffles results in significantly lower IS values (increase the degree of mixing).

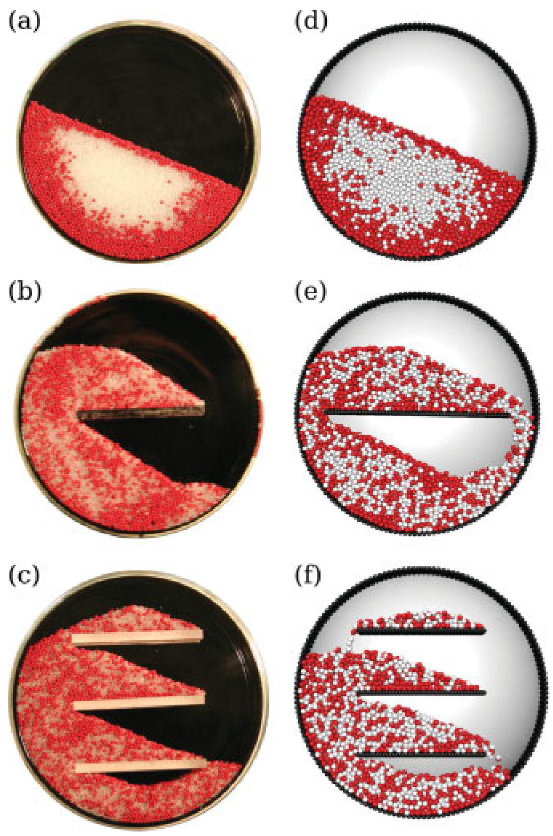

Figure 4. Asymptotic mixing results in tumblers mixers with axially-placed baffles for both experiments (Left) and simulations (Right).

(a)–(d) Unbaffled tumbler, (b)–(e) one axial baffle, and (c)–(f) three axial baffles.

Size segregation

Asymptotic mixing patterns for size segregation systems in tumblers with peripheral baffles are shown in Figure 6. These

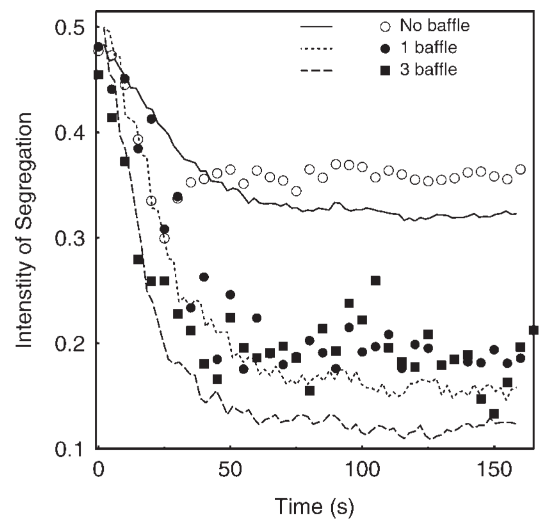

Figure 5. Quantitative mixing results for density segregation in tumbler mixers with axially placed baffles.

Experiments are shown as symbols and simulations as lines. Note that baffles that truncate the flow result in significantly lower IS values (increase the degree of mixing).

results reveal similar trends to those observed for density segregation. The experimental observations and simulations show formation of a core of small particles for the unbaffled and short baffle cases, with only the long baffle case exhibiting reduced segregation.

The quantitative measures of these observations (see Figure 7) again show that while the short traditional baffles produce results similar to the nonbaffled case, the long baffle reduces the measured asymptotic degree of segregation. The dynamics of segregation are also reasonably well predicted by the simulations.

The qualitative comparison for tumblers with axially placed baffles is illustrated in Figure 8. The results in Figure 8

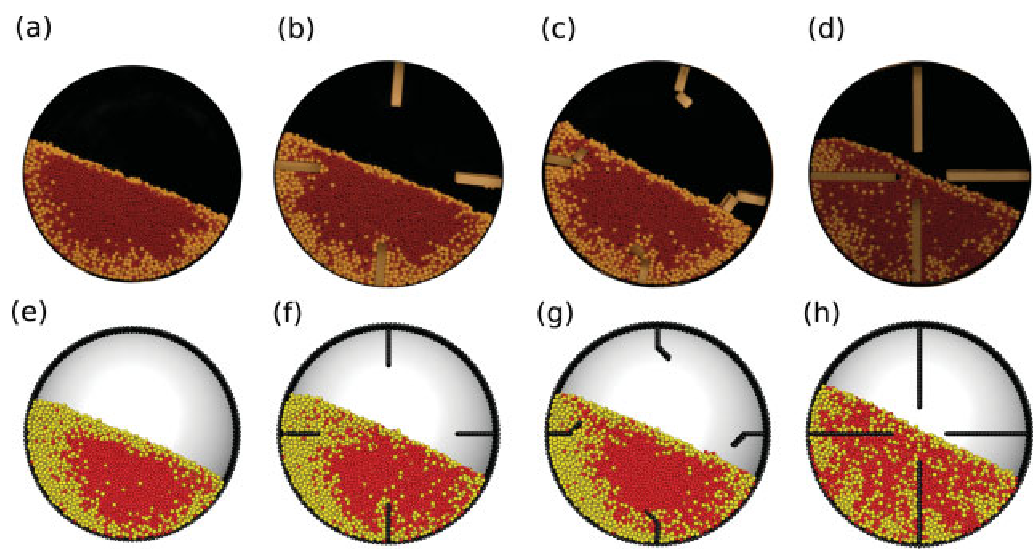

Figure 6. Asymptotic size segregation results in tumblers mixers with baffles at the wall for both experiments (Top) and simulations (Bottom).

(a)–(e) Unbaffled tumbler, (b)–(f) short baffle, (c)–(g) short baffle with angled tip—a traditional configuration, and (d)–(h) long baffles.

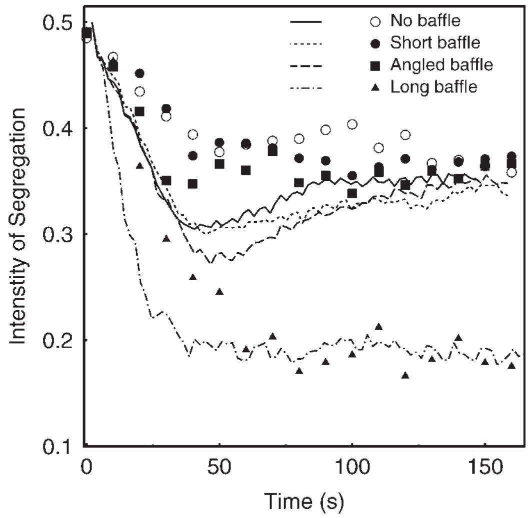

Figure 7. Quantitative mixing results for size segregation in tumbler mixers with baffles attached to the walls.

Experiments are shown as symbols and simulations as lines. Note that unbaffled mixers and mixers with short baffles behave very similarly, while very long traditional baffles results in significantly lower IS values (increase the degree of mixing).

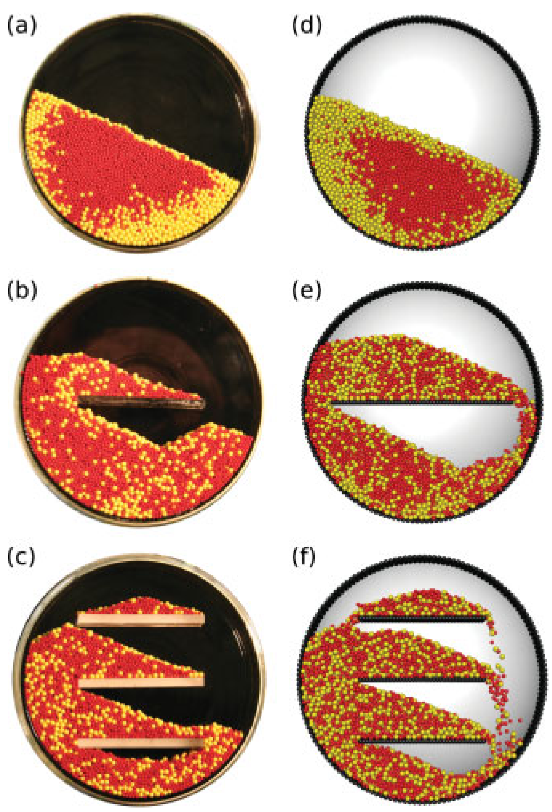

Figure 8. Asymptotic size segregation results in tumblers mixers with axially-placed baffles for both experiments (Left) and simulations (Right).

(a)–(d) Unbaffled tumbler, (b)–(e) one axial baffle, and (c)–(f) three axial baffles.

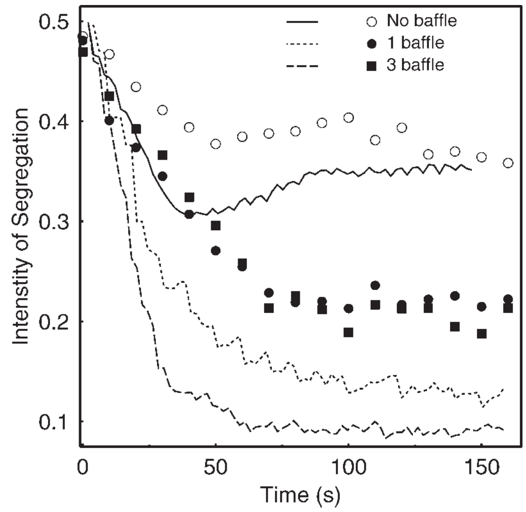

Figure 9. Quantitative mixing results for size segregation in tumbler mixers with axially placed baffles.

Experiments are shown as symbols and simulations as lines.

show the expected observation of core formation in the case of the unbaffled tumbler. Both experiment and simulations show that for axially placed baffles the segregation is highly reduced.

The quantitative data in Figure 9 indicate that placing an axial baffle results in significantly lower IS values. Once again, these results are similar to those observed for density segregation.

The effect of flow inversion on tumbler mixers

We have shown both experimentally and by way of simulations that segregation is dramatically reduced by the placement of axial baffles in a tumbler. We also recognize that this enhancement in mixing is due to the flow modulation introduced by the periodic alteration of the flowing layer. The exact impact of this alteration may be understood by tracking the preferred direction of segregation within these drums, whereby one notes that the static portion of the bed simply "stores" the material and returns it in almost the same orientation for its next pass through the surface layer (i.e., it undergoes a full 180° change in orientation prior to returning to the flowing layer). Toward that end, we begin simulations where we "tag" a vertical line of particles within the flowing layer and denote their original orientation with a black arrow (see frames a and d in Figure 10). In the other frames, the green arrows denote the future average position and orientation of the initially tagged particles as they move through the bed. The broken, red arrow in frame (f) represents a particle orientation that has "folded" upon itself (i.e., the orientation contains a loop from a previous partial layer pass). Note that the orientation of the arrow in the unbaffled case both leaves and enters the flowing layer almost perfectly tangent, while the orientation of the arrow in the baffled case is rotated roughly ninety degrees. The unbaffled case, therefore, results in asymptotically segregated systems even for drums whose surface length L, is small compared to $\bar{u}/t_s$

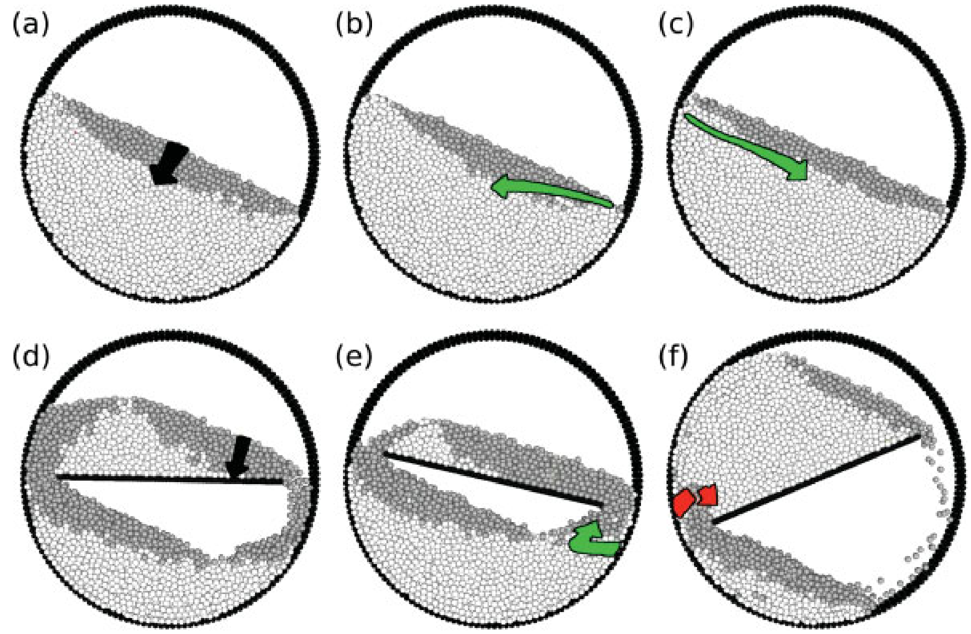

Figure 10. A schematic representation of flow inversion in tumbler mixers.

Images a–f show particle positions from PD simulations for an unbaffled tumbler (a–c), and a tumbler with a single axially-located baffle (d–f).

where $\bar{u}$ is the average velocity within the flowing layer. If we instead place baffles near the axis of rotation, we periodically alter the flowing layer so that we achieve both (1) a smaller average uninterrupted flow length L, and (2) periodic variations in the effective direction of segregation with respect to the tumbler streamlines (as the baffles rotate with the drum; see Figure 10).

The fact that the static bed no longer returns the material to the flowing layer(s) in the same orientation in which it left, is key to the results reported here.

An alternate, yet roughly equivalent, way to analyze the flow is shown in the plot of Figure 11. Recalling that a single pass through the shearing layer would re-orient particles by 180°, one notes that the local segregation orientation will change during the mixing process if a particle passes through the layer, on average, in fewer than one half of a rotation. In order to visualize this, Figure 11a illustrates the distribution of rotations per layer pass in tumblers with axially oriented baffles. Figure 11b presents the results for tumblers with baffles attached to the wall. In these figures, the open symbols represent the unbaffled case, which exhibits a very narrow distribution centered on 0.5 rotations. In contrast, the axially located baffles or long baffles result in much broader multimodal distributions, suggesting that the orientation of particles in future layer passes should be almost uncorrelated with previous passes.

Turning to a practical application of this observation, we note that baffles that induce a broadening in the distribution of particles that undergo a rotation per layer pass are more efficient in suppressing the segregation of mixtures both for size and density. This suggest that regardless of the type of baffles used or their location in the tumbler, the important aspect to keep in mind is that this distribution should be kept as broad as possible. This suggests a simple design principle that is easily tested both experimentally (using tracer particles) or computationally.

Scaling effects

Models to predict segregation in free surface flows are still very basic and applicable only for specific systems.[45] Therefore, predicting the effect of axially oriented baffles for scaling purposes can only be addressed at this time by means of scale-up rules. It has been pointed out by Khakhar et al.[18] that for mixing purposes the details of the velocity field in the shear layer are not important since the asymptotic patterns are insensitive to the details in the velocity field. As shown in the time-modulation section (earlier), the dimensionless critical forcing frequency – which should be less

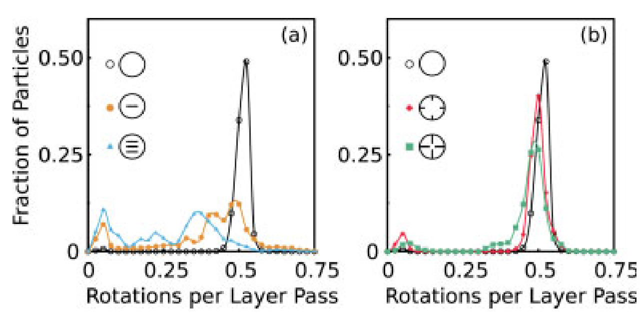

Figure 11. Distribution of rotations per layer pass.

Note that the centrally-located baffles (a), cause broader distributions than traditional baffles (b), although the long traditional baffle yields a more broad distribution than short traditional baffles.

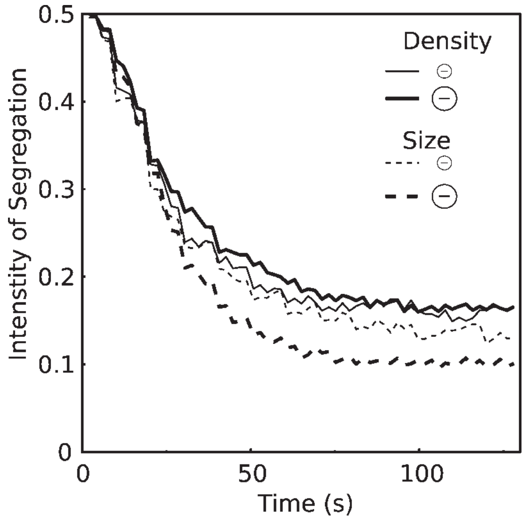

Figure 12. Quantitative mixing results for size and density segregation from PD simulations in tumbler mixers with one axially oriented baffle.

Solid lines illustrate density segregation data in small (thin) and a double sized (thick) tumblers with constant aspect ratio. Dashed lines show results for the size segregation simulations.

than unity to limit segregation – depends on the shear rate, γ, and rotation rate, ω. Based on existing experimental and theoretical observations for unbaffled tumblers and monodispersed systems of particles, Ottino and Khakhar[45] have shown that the shear rate at the midpoint of the layer is given by $\gamma = [g\sin(\beta_m - \beta_s)/(2cR\cos(\beta_s))]^{1/2}$ for all Froude numbers Fr (dimensionless acceleration) and dimensionless tumbler sizes s, where c is a constant. It should be noted that the difference between the dynamic and static angles of repose ($\beta_m - \beta_s$), has been observed to vary with rotation rate as $\omega^{1/2}$,[46] so that the critical dimensionless forcing frequency is a decreasing function of ω and particle radius/diameter, but *does not depend on tumbler size s*. Assuming that these observations hold in a tumbler with one axially-oriented baffle, one would expect that upon scaling this tumbler, keeping the same rotation rate and geometrical aspect ratios for the length of the baffle to radius of the tumbler, the intensity of segregation in the two systems will remain the same at the asymptotic state. As a test of this assertion, in Figure 12, we plot results for both density (solid lines) and size (dashed lines) segregation for tumblers that differ in radius by a factor of two. In both cases, the differences between the small (thin line) and bigger (thick line) systems are quite small, and may be explained by the difference in the number of samples used in the IS calculation.

Conclusions and Perspective

It has been demonstrated that segregation in a rotating tumbler can be dramatically reduced by introducing periodic flow inversions within the drum. Here this is accomplished by introducing an axially placed baffle. Asymptotic values of the intensity of segregation show that this arrangement not only reduces the segregation, but also is able to maintain the mixing over many rotations. Both experimental observations and simulations agree qualitatively and quantitatively for the mixing of initially segregated mixtures of particles that differ in size or density.

One of the main observations that emerge from this study is that although it is generally accepted in the literature that size and density segregation follow different mechanisms of segregation ("percolation" in the first case[47] and "buoyancy" in the second[9]) our approach of flow modulation has the same effect in both types of system. This is significant from a practical standpoint since size and density differences tend to be the most common driving forces for segregation in industrial practice, and, therefore, the solution proposed in this study can be applied to a vast array of processing applications. Although the results presented in this study pertain to a specific example, when combined with those previously reported by Shi et al.[20] on linear shear flows, the approach of flow inversion is valid in general for all free-surface flows.

The results described here raise several issues for further research. As presented in this work, our approach has been tested for size and density segregating systems independent of each other. A combined system in which size and density differences coexist is the natural extension of this research, since these types of systems are the norm in industrial processes. While the approach as described here works for cylindrical containers with 50% filling level at 6RPMs, other geometries, rotation rates, as well as different filling levels require testing. All these issues necessitate further investigation and are the focus of ongoing work.

Acknowledgments

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